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Learning Geometry by The Development of Non-Iconic Spatial Visualization with Dimensional Deconstruction

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Abstract

In geometry, a breach of the didactic contract currently occurs at the time of the primary-secondary transition with regard to spatial visualization. Although iconic visualization (which spontaneously associates a drawing with objects that have been seen and experienced in reality) is practiced in primary education, the acquisition of non-iconic visualization (in terms of a network of straight lines, points etc.) is left entirely up to the student at secondary school. This is without doubt the hardest essential threshold students need to be helped across in the teaching of geometry. The aim of this study was to develop non-iconic visualization in students at the end of primary education in order to facilitate their transition to the theoretical approach to geometry in secondary. The students in the experimental groups showed relative improvements in overall performance that were greater than those of the students in the control groups. The students in the experimental groups were also found to have started to develop visual acuity in the analysis of figures. The results of this study provide guidelines for teachers' work.

Keywords: *Mathematics; Geometry; Didactic; Visualization; Primary/secondary; Learning; Teaching.*

Introduction

The value of geometry teaching during compulsory schooling needs no demonstration. We will therefore confine ourselves to citing a few points from the report of the Commission of Reflection on Mathematics Teaching (2002) [1], which highlights the fact that geometry firstly enables space to be understood by means of, among other things, spatial visualization, and secondly lays the groundwork for learning how to reason. The report also states that geometry is used in everyday life to do things such as follow instructions to assemble a product, change the layout of a space, get around in a building or a city with the help of a map or plan, follow an underground route and so on (Duroisin, 2015) [2]. In professional life, this discipline is equally useful for a number of different jobs, including architecture, engineering and mechanics, as well as for any profession that requires the use of 3D visualization software, for example. In addition, as well as in mathematics, it is also used in other disciplines such as physics and chemistry [3].

However, geometry is one of the most difficult subjects to teach, and one in which students'results are often disappointing [4-10]. Perrin-Glorian (2005, p. 7) [11] argues that 'stubborn difficulties regarding the use of figures in solving geometry problems were identified long ago in secondary school students, both in constructing figures and in demonstrating the properties of geometrical figures'. Despite this, the teaching of geometry in primary school prioritises straight lines, their relationships and their properties and the most obvious 2D shapes (squares, triangle, parallelograms, etc.), the main emphasis being on their outlines. Teaching thus goes no further than a 'surface' view of the figure [12]. Yet, as Duval and Godin note (2005, p. 7) [13], 'such an order of introduction is inconsistent with the way figures are perceived and interpreted outside mathematics'. Moreover, emphasis is placed on a cultural vocabulary (perpendicular, parallel, intersecting, diagonals, medians, lines, rays, segments) and great importance is ascribed to the handling of the traditional instruments only (rulers, set squares, compasses), with a concern for the precision of drawings that takes up much of the teaching time. For her part, Perrin-Glorian (2005) [11] notes that geometry is insufficiently problem-based and that the teaching of it is primarily ostensive. Many difficulties are therefore observed in learning geometry and some of them are specifically linked to visualization.

The research focuses on the development of non-iconic visualization in students at the end of primary education in order to facilitate their transition to the theoretical approach to geometry in secondary. Through this study, the aim is to evaluate the implementation of a didactic engineering that develops dimensional deconstruction. This goal will be pursued by means of activities based on exercises involving completing or reproducing figures. This article presents a quasi-experimentation plan conducted in Belgium and describes his results. The hypothesis is that a didactic transposition focusing on the principle of dimensional deconstruction would enable students in the fourth cycle of primary school gradually to start using visualization of the non-iconic type (in which attention varies between surfaces, lines and points). The independent variable targeted is the development of students' visual acuity in the analysis of figures, while the dependent variable, which will be the subject of observation in the study, concerns the students' performance.

Spatial visualization in geometry

Like mental rotation and spatial orientation, spatial visualization is a spatial ability [14]. This ability can be defined as multistep 'manipulations of spatially presented information' [15]. Lowrie and al. (2019) [14] add that information mentally manipulated are spatial properties of objects. This ability requires flexible activation of different operational strategies [16] and involved cognitive demands of simultaneously encoding and decoding information [17]. For example, imagine the folding and unfolding of a piece of paper is a task that requires spatial visualization [18]. The representation and the manipulatation of images mentally, as the drawing of images, allows encoding graphic information. Decoding is present during interpretation graphic information.

The acquisition of this ability is not spontaneous and required a long and complex learning [19] but can be trained by the way of long or shoter intervention [20-22,14]. Visualization processes play keyrole in geometrical thinking [23]. The development of spatial visualization predicts success in mathematics performance [19,21,24,22,25] and can help student to construct efficient internal representations of problems [13].

The field of geometry in primary school mainly involves visual work on tangible objects, leading to technical work in micro-, meso- and/or macro-space [26,27], and ending up with a conceptualization process that enables reality to be interpreted (Duroisin & Demeuse, 2016) [28]. Duval (2005, p. 6) [29] highlights the fact that 'of all the fields of knowledge that students have to enter, geometry is the one that requires the most comprehensive cognitive activity, requiring manual skills, language and observation. In geometry, one has to construct, reason and see, and the three are inseparable.' It is important to take account of the register of representation, which is still often overlooked, because 'it runs counter to normal cognitive functioning outside mathematics' (Ibid., p. 5). Yet in primary education, the organization of teaching goals in plane geometry leads students to develop knowledge about the properties of objects in one dimension (straight lines, the relationships between straight lines and their properties), and then to work on the familiar two-dimensional shapes (squares, rectangles, triangles etc.), in a somewhat compartmentalised manner, without establishing enough links between the one-dimensional and two-dimensional configurations [30]. This leads to the conclusion, drawn by Duval and Godin (2005, p. 7) [13], that 'students' relationship with the geometric figures is characterised by profound didactic equivocation'. There is also add the difficulty of students to clearly define and understand geometric concepts [31]. Concerning the organization of teaching in this respect, these authors state that 'it comes up against the way the figures are perceived and interpreted outside mathematics. Something that is recognized from the start as a 2D shape cannot be broken down perceptually into a network of 1D shapes' (p. 7). One of the challenges of teaching geometry will therefore be to establish these links, which are too often left entirely to the children, by setting them activities to 'move from a visual analysis of the shapes in terms of assemblages of surfaces (2D shapes) to a visual analysis in terms of assemblages of lines (1D shapes)' [13]. In other words, the aim will be to help students achieve the phenomenon of 'insight', stressed by Gestalt theory: an awareness that enables the subject to overcome the limitations imposed by the shape. This awareness, which runs counter to spontaneous processes of visual identification of shapes, can be practiced using dimensional deconstruction, which requires the development of the capacity to analyze figures visually [32]. It will therefore be necessary to develop in students the geometrically correct way to see a figure through activities designed for this purpose. Duval and Godin (2005, p. 8) [13] assert in this connection that 'without such a transformation of the spontaneous and predominant way of seeing, all formulations of geometric properties are likely to seem meaningless'. This helps us to understand one of the difficulties experienced by students with developing the demonstration method [33]. As for specific activities, Mathé (2012) [34] proposes, as Bouleau (2001) [35] had already suggested, that the artefacts used by the student when completing or reproducing figures (materials promoting 1D visualization such as rulers or set squares and materials promoting 2D visualization such as templates or stencils) should be varied in order to initiate the intended perceptual change : « It is handling the instruments, via the analysis of figures in terms of units of size one or zero that the use of these instruments entails, that changes the way students relate to objects and may enable them to reconcile perceptual analysis with geometric analysis of figures. [...] The teacher can help students to practice these changes in the way of seeing objects by getting them to work with the instruments on exercises in which figures have to be completed [36]. This is similar to the point made by Duval and Godin (2005, p. 8) [13] that 'it is by playing with the variable presented by the instruments in exercises involving reproducing [or completing – ed.] figures that one can overturn the deep-rooted and persistent predominance in students of perceptual analysis over geometric analysis of figures'. The first challenge for adopting a geometric approach is to move from students' usual way of looking at a drawing to the geometrical view of a figure that they need to take [37,12]. In other words, they need to move from iconic visualization (which spontaneously associates a drawing with objects that have been seen and experienced in reality) to non-iconic visualization (in terms of a network of straight lines, points etc.). This is without doubt the hardest essential threshold students need to be helped across in the teaching of geometry.

According to Duval and Godin (2005) [13], there are at least three ways to analyze a figure. The first is natural and occurs in non-mathematical contexts: it is perception. Perception is 'pregnant': it relies on the recognition of forms (or figural units) through their visual properties [38]. While this first type of analysis occurs fairly naturally, the other two are the types of analysis that we seek to develop in students through specially designed activities: « First, there is the knowledge of geometric properties that must be mobilised in response to given hypotheses: geometric properties must then take precedence over visually recognized forms in order to analyze a figure. Second, there is a wide range of instruments that can be used to reproduce or construct a figure: the analysis depends on the reproduction or construction procedures that the instrument requires [13]. These two forms of visualization, which it is desirable to develop in geometry, will make a gradual transition possible from drawing to figure (Figure 1). These authors emphasize that setting students activities involving reproducing or completing figures using various instruments (templates, stencils, information-bearing rulers, compasses, plastic rulers, squares, etc.) to analyze either (1D) figural units or 2D units will enable them to gradually move from 'pregnant' iconic visualization (perceptual analysis) to non-iconic visualization (geometric analysis). In other words, it is the activity that is set, or rather the constraint associated with the use of various instruments (2D or 1D) that determines the student's relationship with the figures.

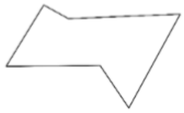
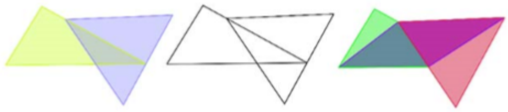
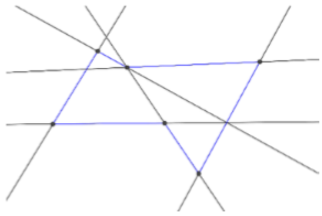
1) 2D drawing	How should it be perceived?	
2) Two visually incompatible decompositions into *2D figural units*	a) Examples of **assembly by juxtaposition**.	
	b) Examples of **assembly by superposition**.	
3) Dimensional deconstruction into *1D or 0D figural units*	Network of **straight lines**: 6 sides that can be prolonged by moving away from the closed outlines. The **points** are obtained here by the intersection of two lines.	

Figure 1: Assembly into 2D, 1D and 0D figural units.

To recognize a shape, two analytical approaches can be used: assembly by juxtaposition (the type that occurs naturally) and assembly by superposition (which is less natural). In the first case, we observe 'as many shapes as there are closed outlines', whereas in the second case the assembly 'visually encourages the extension of the lines recognized as belonging to one shape and not to another' and one observes 'closed outlines rather than shapes' [13]. It is therefore important to stipulate that, quite naturally and according to the individual, one of these two forms of visualization will predominate. Changing this is not easy: it will require the use of specific graphic activities (such as extending the sides of the drawing), the use of templates or coloring.

Another way of understanding this figure is to use dimensional deconstruction. As Bulf indicates (2009, p. 55) [39], this process makes it possible « to bring out relations between smaller figural units and to deduce from them certain geometric properties associated with deductive reasoning related in an axiomatic discourse ». In other words, this involves deconstructing a 2D/2D figure, for example, into smaller figural units (1D or 0D/2D) in order to reveal the relationships between these figural units [40]. This didactic activity involving the extension of segments is of fundamental value, in that it involves gradually moving from a vision of 2D figural units to one of 1D figural units (dimensional deconstruction), thus promoting the transition from surfaces to lines. The extension of the drawing's lines gives it a geometric character which hitherto was not apparent due to its pronounced iconic character; the transition from iconic to non-iconic visualization is thus encouraged. Once the didactic work that enables the student to move to a line-based visualization of figures has been completed, one can then work on visualizing the figures on the basis of points (those at the intersections of the lines that are drawn): these different forms of visualization are of great importance for commencing the process of demonstration.

To visualize a figure, two opposing modes of cognitive functioning can therefore be used: iconic visualization and non-iconic visualization. The first relates more to the profile of the real object (2D vision), while the second corresponds rather to a series of operations that will lead to the recognition of its geometric properties (0D, 1D and 2D vision). As regards iconic visualization, Duval (2005, p. 13) [29] states that it operates according to 'two levels of operations: discriminative form recognition and identification of objects corresponding to the recognized forms' This innate visualization is what is used in daily life, and is based on a strong perceptual potential: one observes shapes, drawings and so on, and attempts to associate them with a known repertoire. Mithalal (2011, p. 114) [41] states that « the subject can only refer to the general form, and cannot perform operations on it without denaturing it [...]. Evidently, this limitation is unacceptable in geometry, as it prevents one from modifying a drawing in order to reveal its properties ».

Consequently, iconic visualization prevents the student from looking at the figures in geometry in the right way and leads to an impasse. According to Mithalal (2011) [41], the subject's perception initially focuses directly on the outline or profile of a figure; accordingly, anything that is outside this field is not, without training, perceived as potentially usable to solve the problem that is set. There is thus a resistance to moving away from the initial perception. In other words, what will first be recognized as a 2D shape is not broken down or decomposed, without training, into a network of 1D figural units. The pregnanz of iconic visualization is also demonstrated by various experiments developed by the Gestaltists that prove that the stimuli implemented in a learning situation are perceived globally [42,43]. For the Gestaltists, learning means arranging and rearranging certain elements differently; it means discovering and developing new relationships between elements which were previously seen as isolated (i.e. developing the non-iconic visualization of subjects). It thus involves solving problems and finding an appropriate solution by restructuring the elements of the situation. In this sense, the subject must play an active role in the learning process. To explain what happens during learning, the Gestaltists refer to the phenomenon of 'insight', illustrated by the experiment in Figure 2. In the situation presented, the discovery of the solution is complicated by the presence of a familiar shape from which it is difficult to break free: the points to be connected are arranged in a square. To find the solution, we need to break free from this form and depart from the limitations it imposes. The term 'insight' thus refers to the awareness that allows the subject to leave behind the limitations imposed by the form (i.e. to gradually acquire non-iconic visualization). This exactly reflects the difficulties explained above regarding the transition from iconic to non-iconic visualization. When a student is asked to solve this problem, he or she remains focused on the outline imposed by the square, making it impossible to complete the set task. This is due to the mode of cognitive functioning of the individual who is guided by the iconic visualization of the forms – the same that he/she uses in non-geometric contexts.

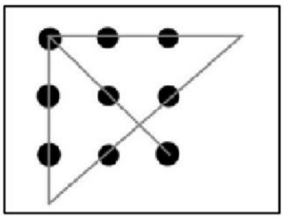


Figure 2: The 'Nine-dot problem', illustrating the phenomenon of insight.

A second impasse lies in the fact that iconic visualization alone can sometimes be misleading and is therefore unreliable: « there may be a conflict between the recognition of forms through their mere resemblance to an archetype and the identification of the object to which the recognized form corresponds » [29]. Thus, many students see a rhombus when a square is shown on its tip or tend to see any quadrilateral as a parallelogram.

A third impasse lies in the fact that « forms are regarded as stable, i.e. they are not initially seen in a way that allows them to be transformed into other similar (or, more importantly, different) forms » [29]. Yet a given figure can generate another if, for example, one visually rearranges the forms that have been recognized and that characterise the initial figure. Figure 3 illustrates this: from the same system of 1D figural units, 2D configurations can be obtained that vary depending on the visual rearrangement that is performed.

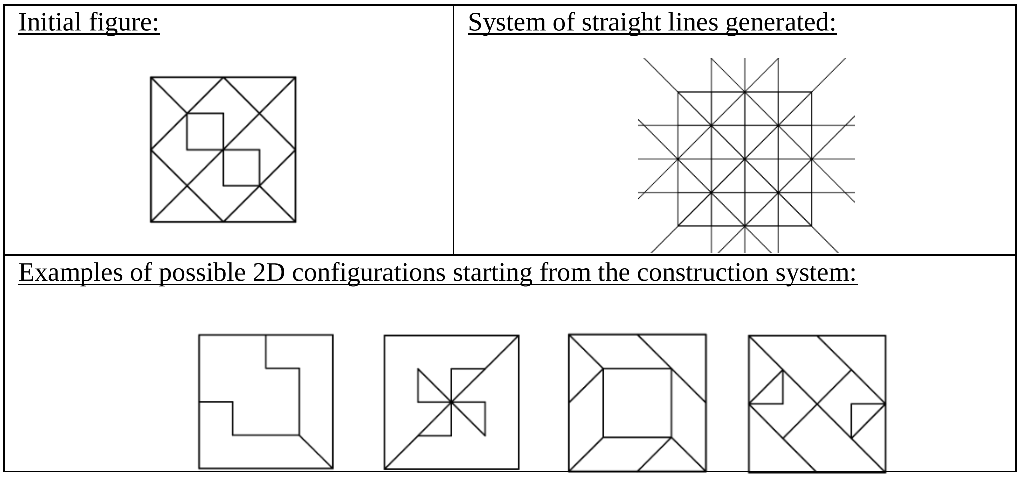


Figure 3: Differing configurations of a figure on the basis of its construction procedure.

A start needs to be made on developing the non-iconic visualization in the fourth cycle of primary education [34] to organize a coherent progression to secondary education and thus avoid a breach of the didactic contract between these two levels of education. To develop primary students' geometric knowledge and enable them to discover some properties of incidence which are essential at the outset of the discipline of geometry, activities based on exercises involving completing or reproducing figures are important; these will require students, by handling instruments, to look in the appropriate geometrical way at the 'model' figure to break it down into 2D, 1D and/or 0D units (i.e. to use dimensional deconstruction). These tasks are nevertheless underestimated [44]. The value of these activities is that they make it easier to grasp the geometric concepts involved through a combination of discourse, visualization and activity and the development of visual acuity in the analysis of figures which is useful for demonstrating the validity of conjectures; in this way, they ease the student's transition to secondary education. Keskessa, Perrin-Glorian and Delplace (2007) [45] and also Mangiante-Orsela and Perrin-

Glorian (2014) [8] confirm the interest of these activities on the visualization development. Nevertheless, several didactic variables play a keyrole in this activity, for exemple geometric instruments used [13,46].

Method

Participants

The experimental sample was of the casual type and consisted of four sixth-grade primary classes (end of cycle 4, before the entry into secondary education) from two separate schools. These schools had a room equipped with an interactive whiteboard (IWB), which was useful for some of the planned teaching sequences. One class from the first school and one class from the second school were the experimental groups. The sampling procedure was non-probabilistic: individuals were selected on the basis of their availability and the teachers' willingness to participate in the experiment. The first two classes each had 20 students while the other two each had 13. Our sample size was therefore 66 students. The schools in which we conducted our experiment were two municipal primary schools under the control of the city of La Louvière (Belgium). The socio-economic indicators for these schools (assigned by the Federation Wallonia-Brussels under the decree of the Government of the Federation Wallonia-Brussels of 24 March 2011) were 5 and 4 out of 20 respectively. The two schools' socio-economic index was quite close, reflecting the homogeneity of the public, which on average was relatively disadvantaged. A request to conduct our experiment in these schools was submitted and agreed to by the municipal board.

Materials and procedure

The object of this study is the development of non-iconic visualization in students through five teaching sequences spread across several 50-minute periods. The complete training module thus corresponds to about twenty hours of lesson time. It was taught from early November to early February. Within the module, the software program GeoGebra® was used on interactive whiteboards (IWBs). This saves the teacher having to draw figures and lines on the board, with the loss of time and lack of precision that this can entail, and allows students to make hypotheses and test and invalidate or confirm them instantly. This is because the program makes it possible to add lines and delete them immediately without having to reconstruct the figure or prompt, but also to construct all the figures which will be used variously as models and prompts for the students. The activities set for the students also aim to get them working on alignments and intersections, in order to bring out properties to do with straight lines and points (properties of incidence, for example). The same teacher, using a complete methodology, taught the experimental treatment to both experimental groups.

During the various sessions offered to students, several didactic variables were used : the types of permitted instruments (templates and stencils making visualization possible in 2D figural units, whereas plasticised rulers, ungradated paper rulers and compasses make visualization possible in 1D figural units), the enlargement or reduction of the figures to be reproduced (to prevent reliance on measurement or in order to see how minor inaccuracies in lines can become very pronounced in an enlarged drawing), differences of orientation (to prevent students from using visual strategies), the introduction of a 'charge' for the use of instruments (encouraging the use of certain instruments rather than others), the type of prompt used (as this affects the knowledge that has to be deployed), and so on. Moreover, the work was carried out over the long term (three months) because, firstly, *a priori* analysis of students' geometry lessons suggested that their non-iconic visualization was not really developed and, secondly, work on dimensional deconstruction had not been started with students in their second educational cycle. Running twelve sessions on this topic would therefore make it possible to ascertain the effects of the independent variable on the dependent variable.

A quasi-experimental design featuring pre- and post-experimental observations combined with a control group would make it possible to observe whether the experimental treatment had any effects on students' performance. First, the four classes were set a pretest without being given any detailed explanation of the purpose of the experiment. The pretest exercises were administered in early November. Five questions were about dimensional deconstruction: a 'model' figure had to be broken down into 2D, 1D and/or 0D units so that it could be reproduced or completed.

Secondly, an experimental treatment was implemented in both experimental groups (i.e. in both classes). The sequences were taught on the basis of the same precise methodology in both groups. The two classes were thus subjected to special treatment: their 33 students received 15 teaching sequences of 150 minutes on the theme of dimensional deconstruction, led by the experimenter. The learning sequences are shown in Table 1. The other two classes (at two different locations), also representing 33 students, pursued the regular curriculum taught by the teacher: thus, they did not receive the experimental treatment. Any reproduction task was proposed to them during this period.

Finally, in a third step, a posttest was taken in mid-February by the four classes using the same procedure and under the same conditions of administration as the pretest.

Table 1: Description of learning sequences taught to experimental groups.

Order	**Subject of Lesson**	**Goals** (At the end of the activity, each student will be able to …)	**Periods**
Sequence 1	Jumbled lines (finding alignments)	- analyze an unfamiliar plane configuration with a view to reproducing it ; - employ relations of incidence such as a point's belonging to a segment, a point's belonging to two segments, intersection, or alignment, as tools for reproducing figure drawings and understanding that these relations may be sufficient, i.e.relations of length do not need to be used ; - establish a relevant chronology of actions (lines to draw) ; - learn to draw precise lines ; - based on the chronology, show that the construction of objects may depend on the construction of other (intermediate) objects.	3 x 50 min
Sequence 2	Introduction to the use of different instruments to complete a figure	- analyze the 'model' figure, draw lines or segments on it that will enable him/her to reproduce it ; - use various techniques and instruments to reproduce or complete a figure ; - employ relations of incidence such as a point's belonging to a segment, a point's belonging to two segments, intersection, or alignment, as tools for reproducing figure drawings ; - establish a relevant chronology of actions (lines to draw) ; - learn to draw precise lines.	6 x 50 min
Sequence 3	The concepts of alignment, straight line and point	- analyze an unfamiliar plane configuration with a view to reproducing it ; - employ relations of incidence such as a point's belonging to a segment, a point's belonging to two segments, intersection, or alignment, as tools for reproducing figure drawings ; - establish a relevant chronology of actions (lines to draw) ; - based on the chronology, show that the construction of objects may depend on the construction of other (intermediate) objects.	2 x 50 min
Sequence 4	Completing figures (using flexible visualization to solve the problem)	- identify the alignment properties of segments and the properties of intersection (i.e. of points) on a given figure ; - use the identified properties to construct straight lines and locate points ; - complete a prompt to complete a model figure, using the simplest method possible.	2 x 50 min
Sequence 5	Completing and reproducing figures	- analyze an unfamiliar plane configuration with a view to reproducing it ; - employ relations of incidence such as a point's belonging to a segment, a point's belonging to two segments, intersection, or alignment, as tools for reproducing figure drawings ; - establish a relevant chronology of actions (lines to draw) ; - based on the chronology, show that the construction of objects may depend on the construction of other (intermediate) objects ; - gradually reduce the laboriousness of such completion tasks.	2 x 50 min

The chosen design also made it possible to control for the pretest effect. The four groups were paired, as the measures were the result of the same subjects passing through all the conditions of the independent variable. Parallel forms of questionnaires were also developed to avoid the problems inherent to test-retest reliability (pretest α = .858, posttest α = .857).

This evaluation device, based on the use of pre- and posttest, also made it possible to calculate raw scores and relative improvements. For each of the two tests, numerical scores relating to the raw results lay between 0 and 30 points. The ten questions (five in the pretest and five in the posttest) were analyzed on the basis of a grid produced *a priori* (Table 2). Six variables were chosen. The coding of the data was dichotomous, as it indicated whether or not the given behaviour was displayed.

Table 2: Variables for coding students' work at pre- and posttest.

1. The student has completed the exercise successfully.
2. The student has drawn on the model.
3. The student has shown some relevant auxiliary lines on the model.
4. The student has shown some relevant auxiliary lines on the prompt (or in the reproduction zone).
5. The student has shown all the relevant auxiliary lines on the model.
6. The student has shown all the relevant auxiliary lines on the prompt (or in the reproduction zone).

The relative improvement was calculated using the following formula: (Posttest score - Pretest score) / (Maximum score - Pretest score) x 100. This was thus the relationship between the student's actual improvement and the maximum that he/she could have improved. As note by D'Hainaut (1985) [47], this index is 'independent of the starting level and since, at the same starting level, it is proportional to performance, we can consider that the relative improvement is proportional to what it is intended to measure' (pp. 158-159).

Results

A priori analysis of the homogeneity of the two groups' results at pretest

Raw scores of overall performances at pretest and posttest were calculated to verify the starting level of the different groups (control and experimental). The performance of the Mann-Whitney test on the two separate groups confirmed this homogeneity (p = 0.301): there was no difference between the results of the different groups in the sample.

Analysis of the results relative to the performance of the learners

Table 3 shows the mean relative improvements in the performance of the control and experimental groups. These means are percentages.

Table 3: Mean relative improvements in overall performance for the two groups of students.

	N	*Mean (%)*	*Standard deviation* Experimental group
Experimental group	33	95.95	9.328
Control group	33	1.98	14.370

As indicated by the data in Table 3, the two groups differed significantly in terms of relative improvement. The group that received the experimental treatment showed a much higher mean than the control group ($p = .0002$). The dispersion of the value of the relative improvement was greater in the control group than in the experimental group.

In order to test students' progress, means corresponding to the success rate obtained for each of the exercises in the two tests (pretest and posttest) were calculated for the students of the two groups. Wilcoxon rank tests on the raw results obtained by the experimental and control groups in each of the pretest and posttest exercises were also carried out for the questions that showed a strong intercorrelation (Table 4). The statistical tests were carried out not on the basis of the means shown in the table, but on the raw scores obtained by each student in each question in the two tests.

Table 4: Wilcoxon rank tests on the pretest and posttest results of the control and experimental groups.

		Control group				Experimental group			
		Mean (%) at pretest	Mean (%) at Posttest	Z	Null hypothesis probability	Mean (%) at pretest	Mean (%) at posttest	Z	Null hypothesis probability
Paires de questions	*1*	6.06	10.1	-1.713	.087	9.09	100	-5.181	.000
	2	13.13	21.21	-1.546	.122	17.68	94.95	-4.936	.000
	3	16.16	14.65	-2.287	.774	18.69	95.45	-5.002	.000
	4	8.08	7.07	-.832	.405	8.08	95.96	-5.117	.000
	5	2.02	5.05	-1.508	.132	3.54	94.95	-5.232	.000

The results of the Wilcoxon tests indicate that there are significant differences in performance between pretest and posttest in favour of the experimental group. The performance differences are very significant in the experimental group for all posttest exercises (p = .000), by contrast with the control group's performance differences, which are not significant (*p* ranging from 0.087 to 0.774).

Study of some of the learners' work

This section presents some of the work done by students at posttest. One question is described here to illustrate the difference between the work by learners from the control and experimental groups. The question (Figure 4) illustrates a figure completion exercises (i.e. a prompt is provided, and the figure has to be completed). Whether the exercise involved reproducing or completing figures, auxiliary lines (such as alignments, diagonals or medians) had to be shown by the student in order to complete the exercise successfully. The use of such lines also showed that the student was gradually moving to a non-iconic visualization type.

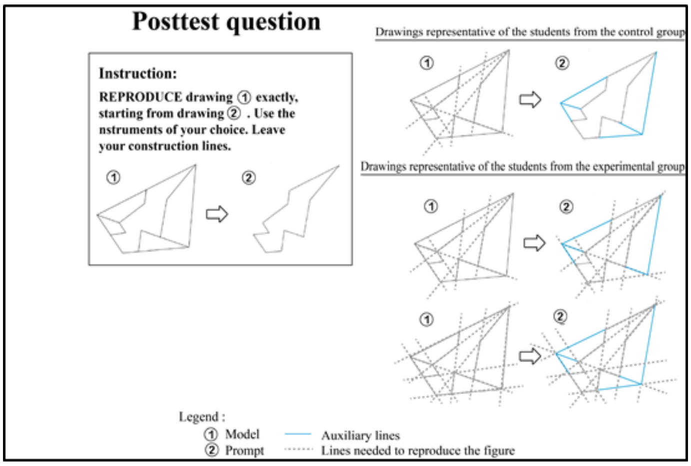


Figure 4: Work by students from the control and experimental groups for another question at posttest.

The Figure 4 illustrates the differences in work between learners from the control and experimental groups for question at posttest. The students from the control group appeared to use neither the model nor the prompt to solve the exercise, whereas the students from the experimental group drew multiple lines on the model① and on the prompt② (Figure 4).

After analyzing the work of all students on this question, the conclusion was clear: only the students from the experimental group drew relevant auxiliary lines on the model and prompt. No student from the control group did so. In addition, it is notable that only 10 students from the control group completed the exercise successfully, against 27 from the experimental group (Figure 5). We can therefore state that the control group students had not developed their non-iconic visualization, unlike the students in the experimental group, in whom this skill was in the process of development.

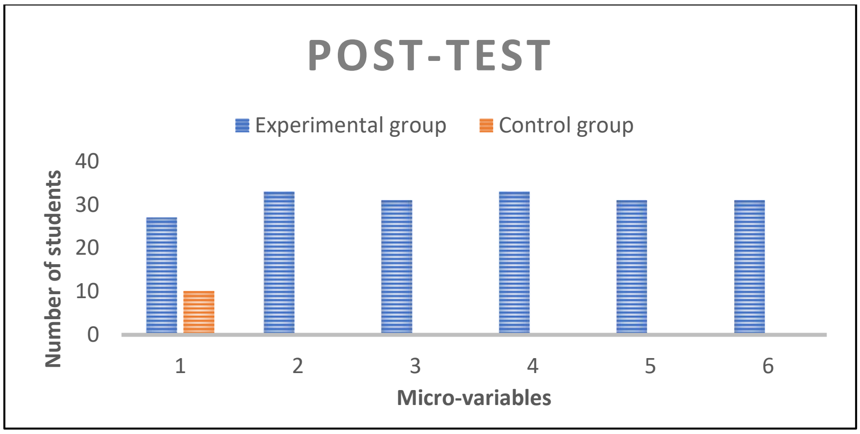


Figure 5: Students' behaviour defined according to six variables (posttest question).

Discussion

The purpose of this study was to observe whether learning sequences that offer problem-solving activities in geometry requiring the use of dimensional deconstruction would allow students to develop their visual acuity in the analysis of figures. The hypothesis was that a didactic transposition focusing on the principle of dimensional deconstruction would allow students in the fourth cycle of primary education gradually to move from an iconic to a non-iconic type of visualization. This transition requires the constant switching of attention between surfaces, lines and points. Being able to adopt 'the right way of seeing things in geometry' is one of the purposes of the discipline, both in order to build the knowledge and skills required at primary level (i.e. the identification of geometric properties such as properties of incidence which, although they seem quite simple at first glance, are rarely mastered by students, cf. Mangiante-Orsola & Perrin-Glorian, 2014) [8] and to facilitate the transition to the theoretical geometric approach in secondary education (i.e. the use of different ways of looking at things introduces them to the process of demonstration).

In order to test the hypothesis, a quasi-experimental design with pre- and post-experimental observations combined with a control group was used. The independent variable in the study was the development of a flexible approach in students in the analysis of the figures, while the dependent variable, which was the subject of observations, concerned students' performance.

First of all, all the students' results at pretest make it clear that learners' non-iconic visualization in their final grade of primary education is weak or non-existent. Very few students used auxiliary lines to enable them to complete or reproduce a figure. In addition, at pretest it was observed that students focused constantly on the closed outlines of the figures [48,49,50]. They did not work spontaneously on the model, because they were unaware that auxiliary lines could be drawn that would help them to complete or reproduce the figures. We can therefore confirm that '[...] acquiring the knowledge needed to analyse a geometric figure is currently left entirely up to the students' (Duval et al., 2005, p. 87) [48]. Moreover, analysis of the geometry classes of the teachers who agreed to participate in the experiment confirmed the finding of Duval and Godin (2005)

[13] that, in primary education, the organisation of teaching goals in plane geometry helps students to develop knowledge of the properties of one-dimensional objects (straight lines, the relationships between lines and the properties of lines) and then to work on familiar two dimensional forms (squares, rectangles, triangles, etc.), in a dissociated manner, without creating enough links between the configurations in one dimension and those in two dimensions.

At the end of the study, the statistical analysis performed revealed the beneficial effect of the experimental treatment on the learners' performance. The overall performance of the students in the control groups did not improve, whereas that of the students in the experimental groups varied very significantly between the results at pretest and posttest, for each exercise performed. The important point here is that, as well as facilitating the primary-secondary transition in geometry, the experimental treatment also allowed students to significantly improve their performance in solving exercises of the kind found in the external certificative tests at the end of the sixth primary grade – an argument in favour of the development of non-iconic visualization in students. This study therefore reinforces the idea that 'We cannot expect that students, who understandably continue with the cognitive functioning associated with iconic visualization, will be able to start understanding statements and discursive processes that rely on non-iconic visualization and that require the visual habit of dimensional deconstruction of forms. Hence the importance of lengthy, specific work to develop these ways of looking at things that are so specific to geometry' [29].

Conclusion

Through this study, it appears important to develop non-iconic visualization in primary students because it does not happen naturally. It is possible by the way of adapted teaching/learning (problem situations, proximal zone of development, scaffolding and spiral teaching, etc.), given that 'completing or reproducing geometric figures allows the knowledge to be built up that is required in primary school, but also enables a relationship to be developed with geometry and the use of instruments that is more consistent with what is expected in secondary school' [51]. After three months of experimental treatment, it is noticeable that students' progress is considerable. Such activities may therefore be introduced in classrooms.

The intention of this approach is to facilitate the transition from primary to secondary education and reduce the breach of the didactic contract between these two levels. Moreover, developping spatial visualization is important to facilitate mathematics learning because spatial visualization and mathematics performance are linked [22,29].

At the very least, this requires teachers to be aware of the issues and objectives of geometry (the importance and usefulness of acquiring non-iconic visualization), and to have received training in teaching this type of spatial visualization.

However, even if research like this is interesting for provide tools to guide the practice of teachers, to hope for a larger change in teaching practices, it is essential that the legal prescriptions (teaching programs) clearly announce recommendations on this subject.

Declarations

Informed consent
The researchers explained the study to the potential students and told them that they were welcome to take part in the study or to choose not to participate. To protect and respect personal data, numbers are used for the students' identification.

Conflict of interest
The authors declare no competing interests.

Availability of data and material
All data and material can be made available to readers

Ethics approval
The authors fulfilled all ethical responsibilities of authors applicable for this journal.

Authors' contributions
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References

1. Commission of Reflection on Mathematics Teaching/Commission de réflexion sur l'enseignement des mathématiques (2002). L'enseignement des sciences mathématiques. In J.P. KAHANE (dir.), *L'enseignement des sciences mathématiques: Rapport au Ministre de l'Education nationale*. Paris: Odile Jacob.
2. Duroisin, N. (2015). *Quelle place pour les apprentissages spatiaux à l'école? Etude expérimentale du développement des compétences spatiales des élèves âgés de 6 à 15 ans.* Thèse de Doctorat, Université de Mons.
3. Celi, V. (2013). Que veut-on que les élèves de l'école primaire apprennent en géométrie*? Actes du XXXXe Colloque COPIRELEM*, Nantes, 110-115.
4. Ambrose, R. & Kenehan, G. (2009). Children's evolving understanding of polyhedra in the classroom. *Mathematical Thinking and Learning*, *11*(3), 158–176. doi:10.1080/10986060903016484.
5. Battista, M. (1999). The importance of spatial structuring in geometric reasoning. *Teaching Children Mathematics*, *6*(3), 170-177.
6. Duval, R. (2014). Linking epistemology and semio-cognitive modeling in visualization. *ZDM: The International Journal on Mathematics Education*, *46*, 159-170.
7. Gavin, M., Casa, T., Adelson, J. & Firmender, J. (2013). The Impact of Challenging Geometry and Measurement Units on the Achievement of Grade 2 Students. *Journal for Research in Mathematics Education*, *44*(3), 478-509.
8. Mangiante-Orsola, C. & Perrin-Glorian, M.-J. (2014). Géométrie en primaire : des repères pour une progression et pour la formation des maîtres. *Actes du XXXXe Colloque COPIRELEM*, Nantes 2013.
9. Michael, P., Gagatsis, A., Avgerinos, E. & Kuzniak, A. (2011). Middle and high school students' operative apprehension of geometrical figures. *Acta Didactica Universitatis Comenianae Mathematics*, *11*, 47-57.
10. Parzysz, B. (1988). "Knowing" vs "seeing", problems of the plane representation of space geometry figures. *Educational Studies in Mathematics*, *19*, 79 – 92.
11. Perrin-Glorian, M.-J. (2005). Des situations pour apprendre à regarder les figures. Quelques résultats et perspectives. *Actes du séminaire national de didactique des mathématiques*, 2004, 71-89.

12. Duval, R. (2011). Idées directrices pour analyser les problèmes de compréhension dans l'apprentissage des mathématiques. *Cuadernos de Investigación y Formación en Educación Matemática*, *11*, 149-161.
13. Duval, R. & Godin, M. (2005). Les changements de regard nécessaires sur les figures, *Grand N*, *76*, 7-27.
14. Lowrie, T., Logan, T., & Hegarty, M. (2019). The Influence of Spatial Visualization Training on Students' Spatial Reasoning and Mathematics Performance. *Journal of cognition and development, 20*(5) 729-751, doi: 10.1080 / 15248372.2019.165329.
15. Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatialability: A meta-analysis.*Child Development, 56*, 138–151.
16. David, L. T. (2012). Training effects on mental rotation, spatial orientation and spatial visualisation depending on the initial level of spatial abilities. Procedia-Social and Behavioral Sciences, 33, 328–332. doi:10.1016/j.sbspro.2012.01.137.
17. Lowrie T. & Logan T. (2018) The Interaction Between Spatial Reasoning Constructs and Mathematics Understandings in Elementary Classrooms. In K. Mix & M. Battista (eds) *Visualizing Mathematics. Research in Mathematics Education* (pp. 253-276). Springer, Cham.
18. Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics, 75*, 191–212.
19. David, L.T. & Clinciu, A.I. (2009). Psychological measures of spatial abilities. Bulletin of the Transilvania University of Braşov, 2(51), 93-98.
20. Wright, R., Thompson, W.L., Ganis, G., Newcombe, N.S. & Kosslyn, S.M. (2008). Training generalized spatial skills. *Psychonomic Bulletin & Review*, *15*(4), 763-771.
21. Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C. & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352-402. doi:10.1037/a0028446.
22. Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Education Psychology, 87*, 170–186. doi:10.1111/bjep.12142.
23. Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education*: Past, present and future inn. 205-236V Rotterdam: Sense.
24. Nagy-Kondor, R. (2014). Importance of Spatial Visualization Skills in Hungary and Turkey: Comparative Studies. *Annales Mathématiques et Informatiques, 43*, 171–181.
25. Hegarty, M. (2018). Ability and sex differences in spatial thinking: What does the mental rotation test really measure? *Psychonomic Bulletin & Review, 25*(3), 1212-1219.
26. Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Dordrecht: Kluwer.
27. Clements, D. H. (2004). Geometric and spatial thinking in early childhood education. In D. H. Clements & J. Sarama (Eds.), & A.-M. DiBiase (Assoc. Ed.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 267–298). Mahwah, NJ: Erlbaum.
28. Duroisin, N., & Demeuse, M. (2016). Le développement de l'habileté de visualisation spatiale en mathématiques chez les élèves âgés de 8 à 14 ans. Petit x, 102, 5-25.
29. Duval, R. (2005). Cognitive conditions of the geometric learning: developing visualization, distinguishing various kinds of reasoning and co-ordinating their running. *Annales de didactique et de sciences cognitives*, *10*, 5-53.
30. Lehrer, R., Jacobson, C., Kemeny, V. & Strom, D. (1999). Building on children's intuitions to develop mathematical understanding of space. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 63–87). Mahwah, NJ: Lawrence Erlbaum Associates.
31. Haj-Yahya, A., Hershkowitz, R., & Dreyfus, T. (2022). Investigating students' geometrical proofs through the lens of students' definitions. *Mathematics Education Research Journal*, 1-27. https://doi.org/10.1007/s13394-021-00406-6.
32. Duval, R. (1995). Geometrical Pictures: Kinds of representation and specific processes. In R. Sutherland & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematical education* (pp. 142-157). Berlin, Springer.
33. Hollebrands, K., Conner, A. & Smith, R. (2010) The Nature of Arguments Provided by College Geometry Students with Access to Technology While Solving Problems. *Journal for Research in Mathematics Education, 41*(4), 324-350.
34. Mathé, A.-C. (2012). Jeux et enjeux de langage dans la construction de références partagées en géométrie. *Recherches en Didactique des Mathématiques, 32*(2), 195-228.
35. Bouleau, N. (2001). Reproduction de figures et géométrie en cycle 1 et 2. *Grand N, 67*, 15-32.
36. Mathé, A.-C. (2008). Confrontation aux objets et processus de conceptualisation en géométrie plane à la fin de l'école primaire, rôle des interactions langagières. *Actes de la Conférence internationale « Efficacité et équité en éducation »*, Université de Rennes 2, 1-14.
37. Duval, R. (2006). A cognitive analysis of problems of comprehension in the learning of mathematics. *Educational Studies in Mathematics*, *61*, 103-131.
38. Coren, S., Porac, C. & Ward, L.M. (1979). *Sensation and Perception*. New-York: Academic Press.
39. Bulf, C. (2009). Analyses en termes d'espaces de travail géométrique sur l'enseignement français de la symétrie en début de collège. *Actes du Premier colloque francochypriote de Didactique des Mathématiques*, 51-70.
40. Mithalal, J. (2010). *Déconstruction instrumentale et déconstruction dimensionnelle dans le contexte de la géométrie dynamique tridimensionnelle*. Thèse de doctorat, Université de Grenoble.
41. Mithalal, J. (2011). Vers la mobilisation d'une géométrie axiomatique et de la déconstruction dimensionnelle : intérêt de la géométrie dynamique tridimensionnelle. *Actes du séminaire national de didactique des mathématiques*, 114-128.
42. Koffka, K. (1935). *Principles of gestalt psychology*. N.Y.: Harcourt, Brace.
43. Köhler, W. (1929). *Gestalt psychology*. N.Y.: Liveright.
44. Bulf, C. & Celi, V. (2015a). Des problèmes de reproduction aux problèmes de restauration de figures plane: quelles adaptations pour la classe? Communication présentée au 41e Colloque COPIRELEM, Mont-de-Marsan.
45.

46. Keskessa, B., Perrin-Glorian, M.-J. & Delplace, J.-R. (2007). Géométrie plane et figures au cycle 3, une démarche pour élaborer des situations visant à favoriser une mobilité du regard sur les figures de géométrie. *Grand N, 79,* 33-60.
47. Barrier, T., Hache, C. & Mathé, A.-C. (2014). Droites perpendiculaires au CM2: restauration de figure et activité des élèves. *Grand N, 93*, 13-37.
48. D'Hainaut, L. (1985). *Des fins aux objectifs de l'éducation* (4è éd.). Bruxelles: Labor-Nathan.
49. Duval, R., Godin, M. & Perrin-Glorian, M.-J. (2005). Reproduction de figures à l'école élémentaire. *Actes du séminaire national de didactique des mathématiques 2004*, 5-89.
50. Godin, M. (2005). De trois regards possibles sur une figure au regard « géométrique ». *Actes du séminaire national de didactique des mathématiques* 2004, 39-70.
51. Duroisin, N., Beauset, R., & Lucchese, J. (23 May 2020). Favoriser le passage à la visualisation non iconique par le recours à une ingénierie didactique pour faciliter la transition primaire/secondaire en géométrie. Annales de Didactique et des Sciences Cognitives, 25 (1), 151-182.
52. Godin, M. & Perrin-Glorian, M.-J. (2008). De la restauration de figures à la rédaction d'un programme de construction. Le problème de l'élève, le problème du maître. *Actes du XXXIVe Colloque COPIRELEM*, Troyes.